# STATISTICAL PHYSICS FOR ANOMALOUS TRANSPORT IN PLASMAS

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The general subject of our group during this year was "***Stable organised motion and coherent structures in fusion plasmas".*** The objective of the research is to emphasize the tendency of self-organisation in plasma in its evolution to stationary regimes as a counterpart of the generally accepted turbulent paradigm. This research is justified by the large set of experimental and numerical evidences showing the intermittent formation of coherent structures and their coexistence with turbulent random phase waves. It is also timely since, for example, the poloidal flows are the origin of high confinement properties in tokamak plasmas and the blob-convective events appear as a systematic transport regime in the divertor region. These are coherent flows under structural instability.

We have investigated:

1. Plasma poloidal rotation (arising from sources distinct of the Reynolds stress and/ or ion-direct loss);

2. The intermittency of the barrier in high-confinement regimes;

3. Formation of coherent trajectory structures due to the trapping produced by the ExB stochastic drift;

4. We have started three new directions (not included in the Work Programme for 2004):

   a) Density pinch and the "snake" phenomenon observed at pellet injection in JET

   b) Vortex coalescence

c) Large Larmor radius effects in the transport of impurities and of fast particles produced by the fusion reaction.

## 1. <u>Plasma poloidal rotation</u>

We have developed our model [1] that is able to correctly represent the full nonlinearity of the ion-temperature-gradient (ITG) driven instability (*barotropic* equation).

$$\beta \frac{\partial \phi}{\partial y} + \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial y} \right) \nabla_\perp^2 \phi - \frac{d^2 U}{dx^2} \frac{\partial \phi}{\partial y} + \varepsilon \left[ \left( -\nabla_\perp \phi \times \hat{n} \right) \cdot \nabla_\perp \right] \nabla_\perp^2 \phi = 0$$

where $\phi$ is the electrostatic potential and U is the poloidal flow. A multiple space-time analysis of the set of ITG equations has led to a simple form, which at stationarity is the Flierl-Petviashvili equation.

$$\nabla_\perp^2 \phi = \alpha \phi - \beta \phi^2$$

We have found an *exact solution* to this equation, which has applications in geophysics, astrophysics, and meteorology. The solution is

$$\phi(x, y) = \frac{\alpha}{2\beta} + s \wp \left( iay + ibx + \omega; g_2 = \frac{3\alpha^2}{(s\beta)^2} \right)$$

where

$$\alpha = \frac{1}{\rho_s^2} \left( 1 - \frac{v_*}{u} \right), \beta = \frac{|e|}{2 m_i u^2} \frac{\partial}{\partial x} \left( \frac{1}{L_n} \right)$$

and $\rho_s$ is the sound Larmor radius, $v_*, u, e, m_i, L_n$ are respectively the diamagnetic velocity, the translation velocity of plasma, the electron charge, the ion mass, the density gradient length. $\wp$ is the double periodic elliptic Weierstrass function and $s$ is a parameter that must be derived from the amplitude of the perturbation fixed either by statistical conditions or by the initial data.

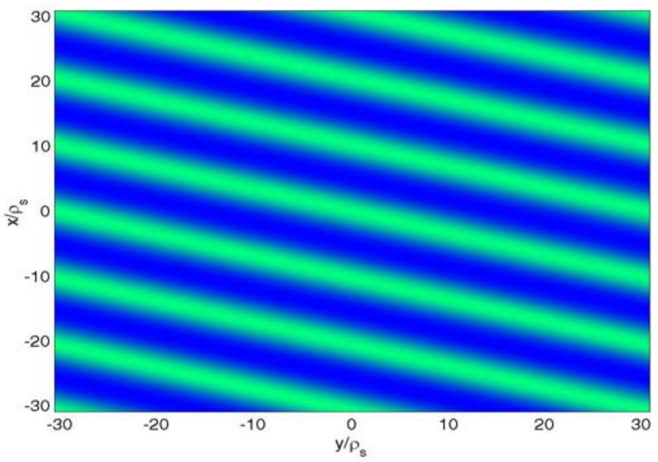


*Figure 1. The electric potential obtained as exact solution of the Flierl-Petviashvili equation.*

This solution has exactly the same form as the *zonal* flow.

The following aspects were examined:

- **Comparisons with experimental data**

We have compared the solution $\phi(x,y)$ with measurements of zonal flows made on Doublet IIID. The agreement is very good: for the width of the periodicity layer; for the shear of the velocity; for the electric field in the radial direction. The agreement is also very good for the measured *spectrum* and we are able to reproduce the symmetrical maxima. We plan to make comparisons also with data from JET.

- **Comparison with numerical simulations**

We have made comparisons with the numerical gyrokinetic simulations from Lausanne. The agreement is very good: for the width of the periodicity layer; for the electric field; slightly higher for us is the maximum of the shear of the velocity.

In conclusion, we have shown in [1] that plasma rotation can be generated at spatial scales intermediate between the ITG eddies and the Larmor radius scales of the Hasegawa-Mima vortices by a nonlinear mechanism determined by the scalar nonlinearity (in contrast to the vectorial or polarization-current nonlinearity). This is an alternative explanation to the existing ones: predator-prey model (P. Diamond), four-wave modulation of ITG eddies (F. Zonca).

We have continued the study of our model for plasma rotation based on the poloidal asymmetry of the turbulence [2]. Plasma rotation can also be generated by a transition induced by a noise in a turbulence that can have multiple scales. It was shown that this process is described by a Langevin equation with several equilibria. These transition processes were analyzed analytically and numerically in collaboration with National Institute for Fusion Research, Nagoya, Japan [3], [4].

## 2. The intermittency of the barrier in high-confinement regimes

We have studied the stability of the zonal flows described by the analytical solution obtained in [1]. The reason for investigating the stability is related with the problem of intermittency events in high confinement regime.

This study is based on analytical methods (we have obtained the dispersion relation of the perturbation around the analytical solution) and on numerical simulation of the evolution of a perturbation of the analytical solution.

- **The stability of the zonal flow (analytical approach)**

We have studied analytically the stability of the solution for zonal flows using two methods:

- one is based on the Naviers-Stokes stability and leads to the Orr-Sommerfeld equation.

- the other is based on the stability of the Kolmogorov flow.

We have studied both methods and we have obtained the dispersion relation, which puts emphasis on the most unstable wavelengths. These have been estimated analytically and then compared with numerical simulations. The agreement is satisfactory (within a factor of 1.5) knowing that the evolution of the vortices becomes rapidly nonlinear. The Orr-Sommerfeld equation is unstable at long wavelength but the selection seems to be related to the periodicity. We have proved that this is the case when we approximate the periodicity such as to reduce the linear equation to Mathieu equation.

We will continue this study by extending the Kolmogorov-flow analysis to include superposition of perturbations located at the neighbor inflection lines.

- **The stability of the zonal flow (numerical approach)**

Our numerical approach to the stability problem is basically oriented to *structural* stability, *i.e.* to the stability to perturbations when the vectorial nonlinearity (of Hasegawa-Mima type, specific to small scale ITG processes) is added to the scalar nonlinearity. We have developed a code of integration (based on libraries NAG and CERN mathematical library) and have done a large series of runs. The results show the tendency of generating vortical structures at a scale that appears to be approximatively $5\rho_s$ .

It still remains to make a detailed statistical analysis to confirm the scale. We also intend to improve the code (to exclude frequent spline interpolations which reduce the precision).

Since this is an important matter we have contacted and started a collaboration with the ENEA Frascati group, in particular Dr. F. Zonca. This is a very useful collaboration and we have found several common subjects (although we keep distinct views on the origin of the zonal flows).

**3. <u>Formation of coherent trajectory structures due to the trapping produced by the ExB stochastic drift</u>**

We have continued the test particle studies of transport and we have shown that the trapping of the trajectories appearing in the ExB drift determines coherence in the stochastic motion and trajectory structures [5], [6].

We have developed a new semi-analytical statistical approach, *the nested subensemble method*. The time dependent diffusion coefficient is determined by means of a set of deterministic trajectories, the average trajectories in subensembles with given values of the potential and of the velocity in their starting point. These trajectories are obtained by dividing each subensemble in a class of subensembles defined by the values of the second derivatives of the potential. Thus, the nested subensemble approach reduces the problem of determining the statistical behaviour of the stochastic trajectories to the calculation of weighted averages of some smooth, deterministic trajectories determined from the Eulerian correlations (EC) of the stochastic potential.

The statistical characteristics of subensembles of trajectories are obtained using this method. We have shown [5] that the statistical behaviour of the trapped trajectories is completely different from that of the free trajectories. The trapped trajectories have a quasi-coherent behaviour. Their average displacement, dispersion and probability distribution function saturate. A very strong anomalous clump effect characterizes neighbouring trapped trajectories. Their clump lifetime is very large compared to the time of flight. This shows that these trajectories form structures similar with fluid vortices. The statistical parameters of these structures (size, build-up time, dispersion) are determined. The trajectories contained in such structures do not contribute to the large time diffusion coefficient. The later is determined by the free trajectories, which have a continuously growing average displacement and dispersion. The probability distribution functions for both types of trajectories are non-Gaussian.

We have also studied the effect of the spectrum of the turbulence on the transport coefficients [7], [8]. We have obtained analytical expressions for the time dependent diffusion coefficient corresponding to given Eulerian correlation of the stochastic potential, using the decorrelation trajectory method. We have shown that in the nonlinear regime characterised by Kubo numbers $K>1$, the scaling law of the diffusion coefficient depends on the shape of the EC and not only on the global parameters of the EC that define the Kubo number. A strong influence has the space-dependence of the EC at large distances, i.e. the small k components of the spectrum. This effect is determined by the complicated process of dynamic trajectory trapping in the structure of the stochastic field. We have obtained a power law scaling in K for the diffusion coefficient in the case of algebraic and Gaussian EC's and we have determined the exponent of the diffusion coefficient as function of the exponent that describes the tail of the EC. It is not a fixed value as in the estimation based on percolation theory but a continuous function that decays from 1 to 0. For more complicated EC of the stochastic potential, the scaling of the diffusion coefficient is not of power law type. Other aspects are studied in [9].

## 4. New subjects

First results were obtained in this period in three new subjects, which were not included in our Work Programme

### 4.1. Density pinch and the “snake” phenomenon observed at pellet injection in JET

There are many experiments on the density pinch but only one significant model, the *turbulent equipartition hypothesis* (Yankov). We propose another view on the physical origin of pinch: if the core can be represented as a collection of vortex lines, there is a condition to embed the “world surface” of each in the three-dimensional space (Gauss-Codazzi equation in the framework of a theory expressed in field-theoretical terms, derived from Nambu-Goto action). This induces a term of mutual interaction, which arises as a self-attraction of the fluid. This has to be evaluated in concrete terms. We have presented this idea in a poster at 31st *EPS Conference on Plasma Physics*, London [10].

### 4.2. Vortex merging and generation of larger scale structures in helicity turbulence

The vortex merging is studied on the basis of a new formulation of the theory. We have proposed for the Hasegawa-Mima (derived from ion convective mode) a formulation in field-theoretical terms (following our previous success of the derivation of the sinh-Poisson equation). We show that a possible transition of merging can be represented with a particular solution, the *sphaleron*. We have presented this incipient stage of this model at *10th EU-US Transport Task Force Workshop*, Varenna [11].

We have studied other aspects of plasma vortices in [12], [13].

### 4.3. Large Larmor radius effects in the transport of impurities and of fast particles produced by the fusion reaction

We have studied the impurity ion transport produced by the Lorentz force in turbulent magnetized plasma. Expressions for the time dependent diffusion coefficient and for the correlation of the Lagrangian drift velocities are obtained in terms of a class of smooth, deterministic trajectories by developing a generalization of the decorrelation trajectory method. This statistical approach is compatible with the invariance of particle energy.

We have shown that the Larmor radius has a strong effect on impurity ion transport in turbulent plasmas [14], [15]. The generally accepted idea that the effective diffusion is reduced due to the cyclotron motion that averages the stochastic potential is not always true. The cyclotron motion can also determine the build up of correlation of the Lagrangian drift velocity by bringing the particles back in the correlated zone of the stochastic potential. The correlation L(t) shows a series of periodic peaks, which lead to increased diffusion coefficients in slowly varying potentials. Consequently, at given Larmor radius, the transport can be reduced or increased, depending essentially on the value of the Kubo number (Figure 2).

These calculations were extended to the range of very large Larmor radii, which characterised the fast particles produced in the fusion reaction. We have shown that even in these conditions the diffusion coefficients can be rather large [16].

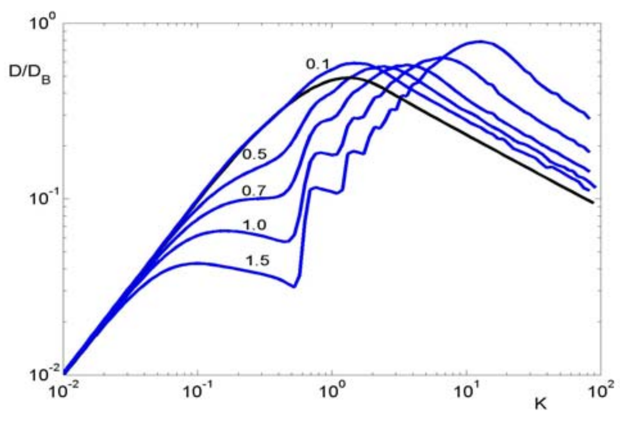


*Figure 2. The diffusion coefficient as function of the Kubo number K for the values of the ration of the Larmor radius to the correlation length that label the curves. The result of the guiding center approximation is the black line.*

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